B.Sc.(Srmester-3)
Subject: Physics
Course: US03CPHY01
Title: Optics

UNIT - II Interference and Diffraction

Interference:

❖ Techniques for Obtaining Interference:

The phase relations between the waves emitted by two independent light sources rapidly changes with time and therefore they can never be coherent. However, if two sources are derived from a single source by some device, then any phase change occurring in one source is simultaneously accompanied by the same phase change in other source.

Therefore, the phase difference between the waves emerging from the two sources remains constant and the sources are coherent. The techniques used for creating coherent sources of light can be divided in to the following two classes.

1. **Wavefront splitting**
2. **Amplitude splitting**

1. Wavefront splitting:

In this methods light wavefront emerging from a narrow slit dividing in to two very closed slits. The wavefront is divided in to two parts. The two parts of the same wavefront travel through different paths and produce fringe pattern. **This is known as interference due to division of wavefront. This method is useful only with narrow sources.** Examples: Young's double slits, Fresnel's double mirror, Fresnel's biprism, Lloyd's mirror etc.

2. Amplitude splitting:

In this method the amplitude (intensity) of a light wave is divided into two parts known as reflected and transmitted components. The two parts travel through different path and returns to produced interference fringes. **It is known as a interference due to division of amplitude.** Examples: Optical elements such as beam splitters, mirrors are used for achieving division of amplitude. Interference in thin film (wedge, Newton's ring etc.), Michelson's interferometer utilize this method.

❖ Fresnel Biprism:

Fresnel used a biprism to show interference phenomenon. The biprism consists of two prisms of very small refracting angles joined base to base. In practice, a thin glass plate is taken and one of its faces is ground and polished till a prism (see figure 1a) is formed with an angle of about 179^0 and two side angles of the order of 30^0.

When a light ray is incident on an ordinary prism, the ray is bent through an angle called the **angle of deviation.** As a result, the ray emerging out of the prism appears to have emanated from a source S' located at a small distance above the real source, as shown in figure 1b. The prism produced a **virtual image** of the source.

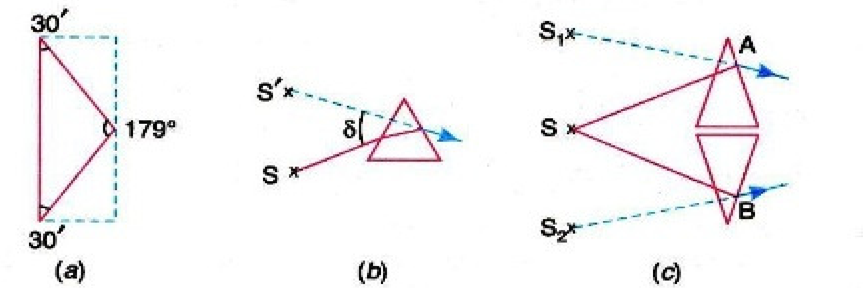


Fig. 1

A biprism creates two virtual sources S_1 and S_2 as shown in fig. 1C. These two virtual sources are images of the same source S produced by refraction and are hence coherent.

❖ Experimental Arrangement:

The biprism is mounted suitably on an optical bench. An optical bench consists of two horizontal long rods, which are kept strictly parallel to each other and at the same level. A monochromatic light source such as sodium vapour lamp illuminates a vertical slit S. Therefore the slits act as a narrow linear monochromatic light source. The biprism is placed in such a way that its refracting edge is parallel to the length of the slit S. A single cylindrical wavefront incident on both prisms. The top portion of wavefront is refracted downward and appear to have emanated from the virtual image S_1. The lower segment, falling on the lower part of the biprism, is refracted upward and appears to have emanated from the virtual source S_2. The virtual sources S_1 and S_2 are coherent as shown in fig.2(a), and hence the light waves are interfere in the region beyond the biprism.

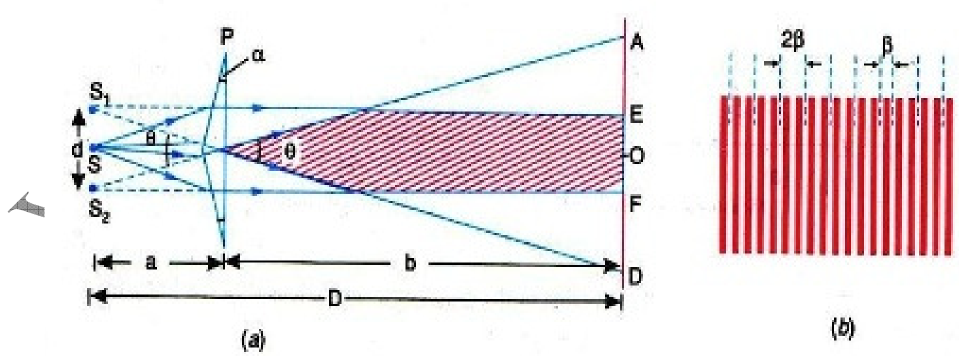


Fig.2

If a screen is held there, interference fringes are seen. In order to observe fringes, a micrometer eyepiece is used.

Theory:

The theory of the interference and fringes formation in case of Fresnel biprism is same as in the case of the double slit. As the point is equidistance from

S_1 and S_2, the central bright fringe of maximum intensity occurs there. On both the sides of O, alternate bright and dark fringes are produced as shown in fig.2(b). The width of the dark or bright fringe is given by

$$\beta = \frac{\lambda D}{d} \qquad (1)$$

Where D = (a+b) is the distance of the source from the eyepiece

❖ Determination of Wavelength of Light:

The wave length of light can be determine by using above equ.(1). The values of β, D and d are to be measured. These measurements are done as follows:

Adjustments:

A narrow adjustable slit S, the biprism, and a micrometer eyepiece are mounted at the same height and in a straight line. The slit is made vertical and parallel to the refracting edges of the biprism by rotating it in its own plane. It is illuminated with the light from the monochromatic source. The biprism is moved along the optical bench till, on looking through it along the axis of the optical bench, two equally bright vertical slit images are seen. Then the eyepiece is removed till the fringes appear in the focal plane of the eyepiece.

(i) Determination of fringe width β :

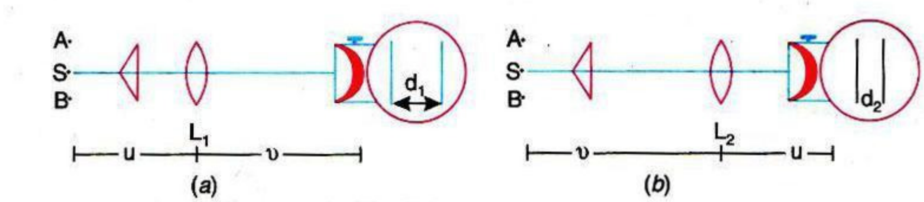
When the fringes are observed in the field of view of the eyepiece, the vertical cross wire is made to coincide with the centre of one of the fringes. The position of the eyepiece is read on the scale, say x_0 .The micrometer screw of the eyepiece is moved slowly and the number of the bright fringes N that pass across the cross-wire is counted. The position of the cross-wire is again read, say x_N.The fringe width is then given by

$$\beta = \frac{x_N - x_0}{N} \qquad (2)$$

(ii) Determination of 'd':

(a) A convex lens of short focal length is placed between the slit and the eye piece without disturbing their positions. The lens is moved back and forth near the biprism till a sharp pair of image of the slit is obtained in the field view of the eyepiece. The distance between the images is measured. Let it be denoted by d_1. Consider figure 3(a), If u is the distance of the slit and v that of the eyepiece from the lens, then the magnification is

$$\frac{v}{u} = \frac{d_1}{d} \qquad (3)$$



Measurement of the distance etween the two virtual sources.

Figure 3

The lens is then moved to a position nearer to the eyepiece, where again a pair of images of the slit is seen. The distance between the two sharp images is again measured. Let it be d_2 . Again magnification is given by

$$\frac{u}{v} = \frac{d_2}{d} \qquad (4)$$

The magnification in one position is the reciprocal of the magnification in the other position. Multiplying the equations (2) and (3) we get

$$\frac{d_1 d_2}{d^2} = 1 \quad or \quad d = \sqrt{d_1 d_2} \qquad (5)$$

Using the value of β, d and D in equation $\beta = \frac{\lambda D}{d}$ The wavelength λ can be computed

(b) **The value of d can be determined as follows**:

The deviation δ produced in the path of a ray by a thin prism is given by

$$\delta = (\mu - 1)\alpha$$

Where α is the refracting angle of the prism. From fig.2(a) it is seen that $\delta = \theta/2$, Since d is very small, we can write $d = a \cdot \theta$

$$\frac{\theta}{2} = \frac{d}{2a} = (\mu - 1)\alpha$$

$$d = 2a(\mu - 1)\alpha \qquad (6)$$

Interference Fringes with White Light:

In the biprism experiment if the slit is illuminated by white light, the interference pattern consists of a central **white fringe** flanked on its both the sides by a few coloured fringes. The central white fringe is the **zero-order** fringe. With monochromatic light all the bright fringes are of the same colour and it is not possible to locate the zero order fringe. Therefore, in order to locate the zero order fringe the biprism is to be illuminated by white light.

Lateral Displacement of Fringes:

The biprism experiment can be used to determine the thickness of a given thin sheet of transparent material such as glass or mica. If a thin transparent sheet is introduced in the path of one of the two interfering beams, the fringe gets displaced from the central fring. By measuring the amount of displacement, the thickness of the sheet can be determined.

Suppose S_1 and S_2 are the virtual coherent monochromatic sources. The point O is equidistant from S_1 and S_2. Therefore, the optical path $S_1O = S_2O$. Let a transparent plate G of thickness t and refractive index μ be introduced in the path of one of the beams as shown in fig.4, The optical path lengths S_1O and S_2O are now not equal and the central bright fringe shifts to P from O. The light waves from S_1 to P travel partly in air and partly in the sheet G. The distance travel in air is$(S_1P - t)$ and that in the sheet is t.

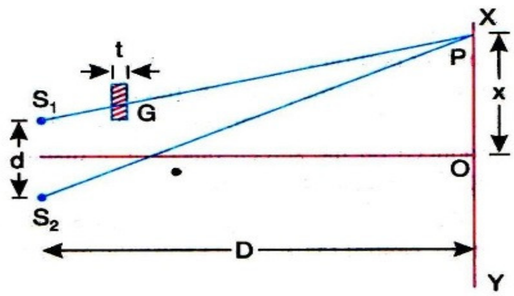


Figure 4

The optical path Δ_{S_1P}= $(S_1P - t) + \mu t = S_1P + (\mu - 1)t$

The optical path Δ_{S_2P}= S_2P

The optical path difference at P is $\Delta_{S_1P} - \Delta_{S_2P}$= 0, since in the presence of the thin sheet the optical path lengths S_1P and S_2P are equal and central zero fringe is obtained at P.

Therefore, Δ_{S_2P}= S_2P

$$S_1P + (\mu - 1)t = S_2P$$

$$S_2P - S_1P = (\mu - 1)t$$

But the path difference $S_2P - S_1P = \frac{xd}{D}$ where x is the **lateral shit** of the central fringe due to the introduction of the thin sheet.

$$(\mu - 1)t = \frac{xd}{D}$$

Thus the thickness of the sheet is given as

$$t = \frac{xd}{D(\mu - 1)} \qquad (7)$$

❖ Lloyd's Single Mirror:

In 1834, Lloyd devised an interesting method of producing interference, using a single mirror. The Lloyd's mirror consists of a plane mirror about 30cm in length and 6 to 8 cm in breadth as shown in fig.5

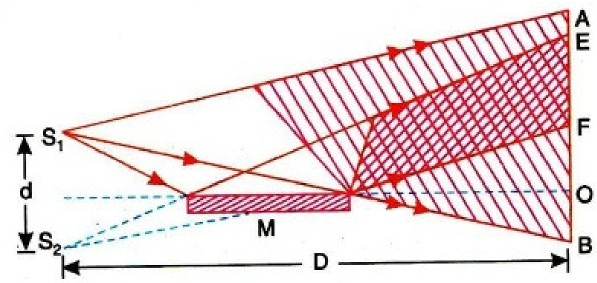


Figure 5

It is polished on the front surface and blacked at the back to avoid multiple reflections. A cylindrical wavefront coming from a narrow slit S_1 falls on the mirror which reflects a portion of the incident wavefront, gives a virtual image of the slit S_2. Another portion of the wavefront proceeds directly from the slit S_1 to the screen. The slits S_1 and S_2 act as two coherent sources. Interference between direct and reflected waves occurs within the region of overlapping of the two beams and fringes are produced on the screen placed at a distance D from S_1 in the shaded portion EF.

The point O is equidistant from S_1 and S_2. Therefore central (zero-order) fringe is expected to lie at O and it is also expected to be bright. However it is not usually seen, since the point O lies outside the region of interference.

By moving the screen nearer to the mirror such that it comes into contact with the mirror, the point O can be just brought into the region of interference. With white light the central fringe at O is expected to be white but in practice it is dark. The occurrence of dark fringe is due to change of phase π when reflected from the mirror. The phase change of π equal to a path difference of λ/ 2 and hence destructive interference occurs.

Determination of Wavelength:

The fringe width is given by $\beta = \frac{\lambda D}{d}$

Measuring β, D and d the wavelength λ can be determined.

Comparison between the fringes produced by biprism and Lloyd's mirror:

Sr No	Biprism	Lloyd's mirror
1	The complete set of fringes is obtained	Only few fringes on one side of the central fringe are observed and the central fringe being itself invisible
2	The central fringe is bright	The central fringe is dark
3	The central fringe is less sharp	The central fringe is sharp

❖ Newton's Ring:

Newton's rings are an example of fringes of equal thickness. Newton's rings are formed when a plano-convex lens P of a large radius of curvature placed on a sheet of plane glass AB is illuminated from the top with monochromatic light as shown in fig. 6

The combination forms a thin circular air film of variable thickness in all directions around the point of contact of the lens and the glass plate. The air film is circular centre is O. The interference fringes are observed in the form of a series of concentric rings with their centre at O. Newton originally observed these concentric circular fringes and hence they are called **Newton's rings**

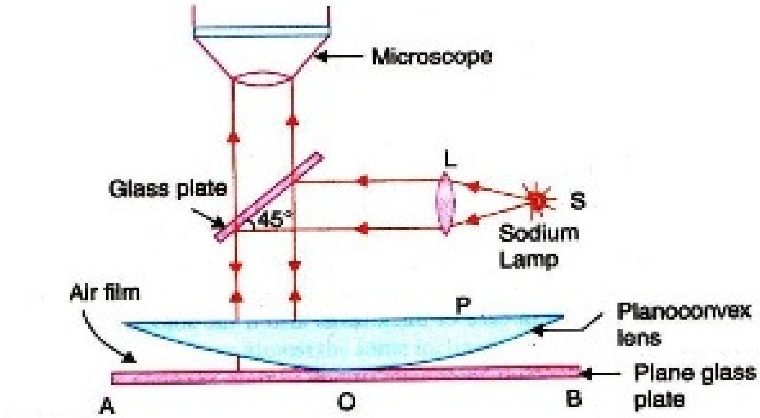


Figure 6

The experimental arrangement for observing Newton's rings is shown in fig.6. The light from monochromatic source is incident on convex lens L, the rays becomes parallel and it is incident on a glass plate inclined at 45^0 to the horizontal, and is reflected normally down on to a plano-convex lens placed on a flat glass plate. Part of the light incident on the system is reflected from the glass –to-air boundary, say from point D as shown in fig.7.

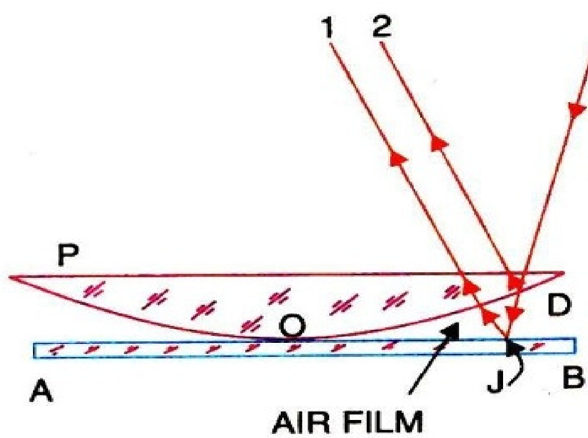


Figure 7

The remainder of the light is transmitted through the air film. It is again reflected from the air-to-glass boundary, say from point J. The two rays reflected from the top and bottom of the air film are divided through **division of amplitude** from the same incident ray CD and are therefore coherent. The rays 1 and 2 are close to each other and interfere to produce brightness or darkness depends on the path difference between the two reflected light rays, which in turn depends on the thickness of the air film at the point of incidence.

Conditions for Bright and Dark Rings:

The optical path difference between the rays is given by

$$\Delta = 2\mu t \cos r - \lambda/2$$

Since $\mu = 1$ for air and $\cos r = 1$ for normal incidence of light,

$$\Delta = 2t - \frac{\lambda}{2}$$

- The intensity maxima occur when the optical path difference $\Delta = m\lambda$

If the difference in the optical path between the two rays is equal to an **integral number of full waves,** then the rays meet each other **in phase**. The crests of one wave falls on the crests of the other and the waves **interference constructively**.

Thus, if $2t - \frac{\lambda}{2} = m\lambda$ or

$\mathbf{2t = (2m+1)\lambda/2}$ **, then bright fringe is obtained.**

- The intensity minima occur when the optical path difference is $\Delta = \frac{(2m+1)\lambda}{2}$

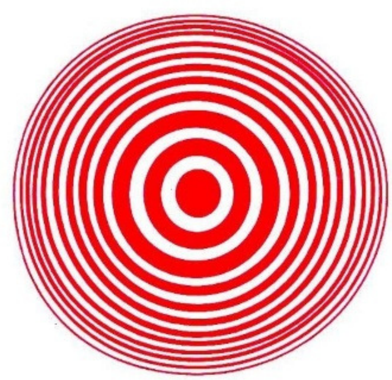
If the difference in the optical path between the two rays is equal to **an odd integral number of half-waves**, then the rays meet each other in **opposite phase**. The crest of one wave fall on the troughs of the other and **the waves interfere destructively**.

Hence if, $2t - \frac{\lambda}{2} = \frac{(2m+1)\lambda}{2}$, or

$\mathbf{2t = m\lambda}$**, then dark fringe is produced.**

Circular Fringes:

In Newton's ring arrangement, a thin air film is enclosed between a plano-convex lens and a glass plate. The thickness of the air film at the point of contact is zero and gradually increases as we move outward. The locus of points where the air film has the same thickness then fall on a circle whose centre is the point of contact. Thus, the thickness of air film is constant at points on any circle having the point of lens –glass plate contact as the centre. The fringes are therefore circular.



Determination of wavelength of Light:

A plano-convex lens of large radius of curvature say about 100cm, and a flat glass plate are cleaned. The lens is kept with its convex face on the glass plate. The system is held under a low power traveling microscope kept before a sodium vapour lamp. It is arranged that the yellow light coming from the sodium lamp falls on a glass plate held at 45^0 light beam. The light is turned through 90^0 and is incident normally on the lens-plate system. The microscope is adjusted till the circular rings came in to focus. The centre of the cross wire is made to come into focus on the centre of the dark spot, which is at the centre of the circular ring system.

Now turning the screw the microscope is moved on the carriage slowly towards one side , say right side. As the cross wire move in the field of view, dark rings are counted. The movement is stopped when the 20^{nd} dark ring is reached. The vertical cross wire is made tangential to the 20^{th} ring and the reading is noted with the help of the scale graduated on the carriage. Thus starting from the 20^{th} ring , the tangential position of the 19^{th},18^{th} , 17^{th}, 16^{th}, ...5^{th} dark rings are noted down. Now the microscope is moved quickly to the left side of the ring system and it is stopped at the 5^{th} dark ring. The cross wire is again made tangential to the 5^{th} dark ring and its position is noted. Thus starting from the 5^{th} ring , the tangential position of the 6^{th},7^{th} , 8^{th}, 9^{th}, ...20^{th} dark rings are noted down.

The difference between the readings on right and left sides of the 5^{th} dark ring gives the diameter value. The procedure is repeated till 20^{th} ring is reached and its reading is noted. From the value of the diameters, the squares of the diameters are calculated. A graph is plotted between D_m^2 and the ring number "m". A straight line graph would be obtained as shown in figure 8.

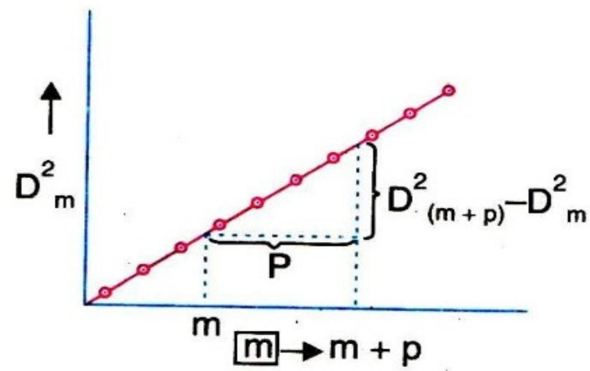


Figure 8

We have,

$$D_m^2 = 4m\lambda R \qquad for\ m^{th}\ ring$$

and

$$D_{m+p}^2 = 4(m+p)\lambda R \qquad for\ (m+p)^{th}\ ring$$

$$D_{m+p}^2 - D_m^2 = 4p\lambda R$$

$$\lambda = \frac{D_{m+p}^2 - D_m^2}{4pR}$$

$$\lambda = \frac{slope}{4R}$$

The slope of the straight line in figure 8 gives the value of 4λR .

The radius of curvature R of the lens may be determined using a spherometer and λis computed with the help of the above equation.

❖ Multiple Beam Interferometry-
Multiple Reflections from a Plane Parallel Film:

The high order reflection occurring at interfaces of thin film are negligible. But, if for any reason the reflection of the interface is not negligible, then the higher order reflections are to be taken into account. When the reflected or transmitted beams meet, multiple beam interference takes place. We are especially interested in the fringes associated with the air space between two reflecting surfaces.

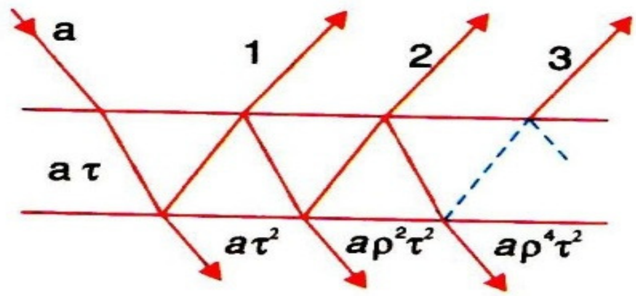


Figure 9

Let us consider the reflected rays 1,2,3, etc. as shown in figure 9. The amplitude of the incident ray is a. Let ρ be the reflection coefficient, τ the transmission coefficient.

The amplitude coefficient of reflection is

$$\rho = \frac{amplitude\ of\ the\ reflected\ wave}{amplitude\ of\ the\ incident\ wave}$$

If the film does not absorb light, the amplitudes of the reflected and transmitted waves are ρ and $a(1-\rho)$ respectively.

Intensity Distribution:

Let a be the amplitude of the light incident on the first surface. A certain fraction of this light aρ, is refracted and another fraction aτ is transmitted see figure 9. The factors ρ and τ are known as the amplitude reflection coefficient and amplitude transmission coefficient respectively. Again at the second surface, part of the light is refracted with amplitude $a\rho^2$ and part is transmitted with amplitude $a\tau^2$. The next ray is transmitted with an amplitude $a\rho^2\tau^2$, the next one with after that with $a\rho^4\tau^2$ and so on. If T and R be the fractions of the incident light intensity which are respectively transmitted and refracted at each silvered surface, then τ^2 = T and ρ^2 = R . Therefore the amplitude of the successive rays transmitted through the pair of plates will be

aT, aTR, aTR^2, ……..

In complex notation , the incident amplitude is given by $E = ae^{i\omega t}$. Then the waves reaching a point on the screen will be

$$E_1 = aTe^{i\omega t}$$

$$E_2 = aTRe^{i(\omega t-\delta)}$$

$$E_3 = aTR^2e^{i(\omega t-2\delta)}, and\, so\ on$$

$$E_N = aR^{(N-1)}Te^{j[\omega t-(N-1)\delta]}$$

By the principle of superposition, the resultant amplitude is given by

$$A = aT + aTRe^{-i\delta} + aTR^2e^{-2i\delta} + aTR^3e^{-3i\delta} + \ldots\ldots \tag{8}$$

$$A = aT[1 + Re^{-i\delta} + R^2 e^{-2i\delta} + R^3 e^{-3i\delta} + ...]$$

Using this expression for sum of the terms of a geometrical progression, we get

$$\text{A} = aT\frac{1 - R^N e^{-iN\delta}}{1 - Re^{-i\delta}} \qquad (9)$$

When the number of terms in the above expression approaches infinity, the term R^N $e^{-iN\delta}$ tends to zero, and the transmitted amplitude reduces to

$$A = aT[\frac{1}{1 - Re^{-i\delta}}]$$

The complex conjugate of A is given by

$$A^* = aT[\frac{1}{1 - Re^{+i\delta}}] \qquad (11)$$

The transmitted energy I_T = AA^*

$$= \frac{a^2T^2}{(1 - \text{Re}^{-i\delta})(1 - \text{Re}^{+i\delta})} \qquad = \frac{a^2T^2}{1 + R^2 - R\,(e^{i\delta} + \text{e}^{-i\delta})}$$

$$= \frac{a^2T^2}{1 + R^2 - 2R\,Cos\delta} \qquad = \frac{a^2T^2}{1 + R^2 - 2R + 2R - 2RCos\delta)}$$

$$= \frac{a^2T^2}{(1 - R)^2 + 2R(1 - Cos\delta)} \qquad = \frac{a^2T^2}{(1 - R)^2 + 4R\,Sin^2\frac{\delta}{2}}$$

$$= \frac{a^2T^2}{(1 - R)^2}\left[\frac{1}{1 + \frac{4R}{(1 - R)^2}Sin^2\frac{\delta}{2}}\right] \qquad (12)$$

The intensity will be maximum when $\sin^2\frac{\delta}{2}$ =0 , i.e. δ= 2mπ.

Where m= 0,1,2,3,4,5,.

Thus, $$I_{\max} = [\frac{a^2T^2}{(1 - R)^2}] \qquad (13)$$

The intensity will be a minimum, when $\sin^2\frac{\delta}{2}$ =1 i.e. δ =(2m+1)π,

Where m= 0,1,2,3,4,5,

$$I_{\min} = [\frac{a^2T^2}{(1 - R)^2} \cdot \frac{1}{1 + \frac{4R}{(1 - R)^2}} = \frac{a^2T^2}{(1 + R)^2}] \qquad (14)$$

We can now rewrite the equation (12) as

$$I_T = \frac{I_{\max}}{1 + \frac{4R}{(1 - R)^2}Sin^2\frac{\delta}{2}} \qquad (15)$$

Similarly the interference intensity from the reflected light beams can be shown to be

$$I_R = \frac{4RSin^2(\frac{\delta}{2})I_{max}}{(1-R)^2 + 4RSin^2(\frac{\delta}{2})} \qquad (16)$$

❖ Fabry- Perot Interferometer and Etalon

The Fabry- Perot Interferometer is a high resolving power instrument, which makes use of the fringes of equal inclination, produced by transmitted light after multiple reflections in an air film between the two parallel highly reflecting glass plates.

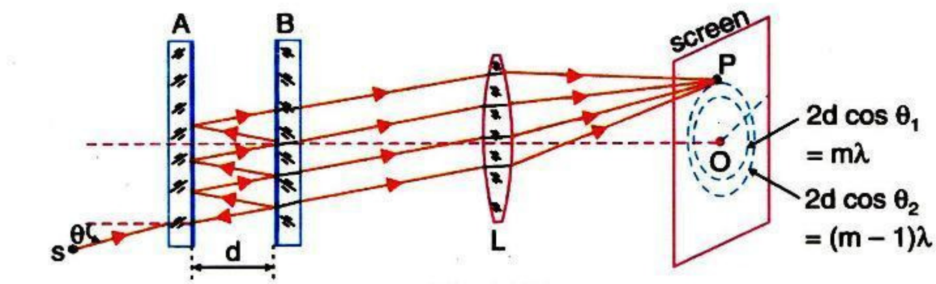


Fig. 11

As shown in figure 11, the interferometer consists of two optically plane glass plates A and B with their inner surfaces silvered, and placed accurately parallel to each other. Screws are provided to secure parallelism if disturbed. This system is difficult to manufacture and is no more in use. Instead of it an etalon which is more easily manufactured is used.

The etalon consists of two semi-silvered plates rigidly held parallel at a fixed distance apart. The reflectance of the two surfaces can be as high as 90 to 99%. Although both reflected and transmitted beams interfere with each other, the Fabry-Perot interferometer is usually used in the transmissive mode.

S is a broad source of monochromatic light and L_1 convex lens, which is not shown in the figure, but which makes the rays parallel. An incident ray suffers a large number of internal reflections successively at the two silvered surfaces see figure11. At each reflection a small fraction of light is transmitted also. Thus each incident ray produces a group of coherent and parallel transmitted rays with a constant path difference between any two successive rays. A second convex lens L brings these rays together to a point in its focal plane where they interfere. Hence the rays from all points of the source produce an interference pattern on a screen placed in the focal plane of the lens.

Formation of Fringes:

Let d be the separation between the two silvered surfaces and θ the inclination of particular ray with the normal to the plates. The path difference between any two successive transmitted rays corresponding to the incident ray is $2d\cos\theta$, the condition for these rays to produce maximum intensity is given by

$$2d\cos\theta = m\lambda \qquad (20)$$

Here m is an integer. The locus of point in the source, which give rays of constant inclination, θ is a circle. Hence, with an extended source, the interference pattern consists of a system of bright concentric rings on a dark back ground, each ring corresponding to a particular value of θ.

Determination of Wavelength:

When the reflecting surface A and B of the interferometer are adjusted exactly parallel, circular fringes are obtained. Let m be the order of the bright fringe at the centre of the fringe system. As at the centre θ=0, we have

$$2t = m\lambda$$

If the movable plate is moved a distance λ/ 2, 2t changes by λ and hence a bright fringe of the next order appears at the centre. If the movable plate is moved from the position x_1 to x_2 and the number of fringes appearing at the centre during this movement is N then

$$N\frac{\lambda}{2} = x_2 - x_1 \; or \lambda = \frac{2(x_2 - x_1)}{N}$$

Measuring x_1 , x_2 and N one can determine the value of λ

Measurement of Difference in Wavelength:

The light emitted by a source may consist of two or more wavelengths, D1 and D2 lines in case of sodium. Separate fringe patterns corresponding to the two wavelengths are not produced in Michelson interferometer. Hence, **Michelson interferometer is not suitable to study the fine structure of spectral lines**. On the other hand, in Fabry- Perot interferometer, each wavelength produces its own ring pattern and the patterns are separated from each other. Therefore, **Fabry-Perot interferometer is suitable to study the fine structure of spectral lines**.

Difference in wavelengths can be found using coincidence method. Let λ_1 and λ_2 be two very close wavelengths in the incident light. Let us assume that $\lambda_1 > \lambda_2$. Initially, the two plates of the interferometer are brought into contact. Then the rings due to λ_1 and λ_2 coincide partially. Then the movable plate is slowly moved away such that the ring systems separate and maximum discordance occurs. Then the rings due to λ_2 are half way between those due to λ_1 . Let t_1 be the separation between the plates when maximum discordance occurs. At the centre

$$2t_1 = m_1 \lambda_1 = (m_1 + \frac{1}{2})\lambda_2 \qquad (21)$$

$$\text{or} \quad m_1 (\lambda_1 - \lambda_2) = \frac{\lambda_2}{2}$$

$$m_1 = \frac{\lambda_2}{2(\lambda_1 - \lambda_2)} \qquad (22)$$

Using this value of m_1 in equation (21), we get

$$2t_1 = \frac{\lambda_2}{2(\lambda_1 - \lambda_2)}\lambda_1 \qquad (23)$$

also

$$\lambda_1 - \lambda_2 = \frac{\lambda_1\lambda_2}{4t_1} \cong \frac{\lambda^2{}_{mean}}{4t_1} \qquad (24)$$

$$but, \lambda_1\lambda_2 = \lambda^2{}_{mean} and \quad \lambda_1 - \lambda_2 is very\ small$$

When the separation between the plates is further increased, the ring systems coincide again and the separate out and maximum discordance occurs once again. If t_2 is the thickness now,

$$2t_2 = m_2\lambda_1 = (m_2 + \frac{3}{2}\lambda_2) \qquad (25)$$

From equations (25) and (21) we get

$$2(t_2 - t_1) = (m_2 - m_1)\lambda_1 = (m_2 - m_1)\lambda_2 + \lambda_2 \qquad (26)$$

$$or \qquad (m_2 - m_1) = \frac{\lambda_2}{(\lambda_1 - \lambda_2)}$$

Using the above expression into equation (26) we get,

$$2(t_2 - t_1) = \frac{\lambda_1\lambda_2}{(\lambda_1 - \lambda_2)}$$

$$\therefore (\lambda_1 - \lambda_2) = \frac{\lambda_1\lambda_2}{2(t_2 - t_1)} = \frac{\lambda^2{}_{mean}}{2(t_2 - t_1)} \qquad (27)$$

❖ Diffraction:

Diffraction is the bending of waves around the edges of an obstacle is called diffraction. When the waves encounter obstacles or openings , they bend round the edge of the obstacles , if the dimensions of the obstacles are comparable to the wavelength of the waves. The bending of waves around the edges of an obstacle is called **diffraction.**

Diffraction is the result of interaction of light coming from different parts of the same wave front. The different maxima are of varying intensities with maximum intensity for central maxima. Diffraction fringes are not of the same width.

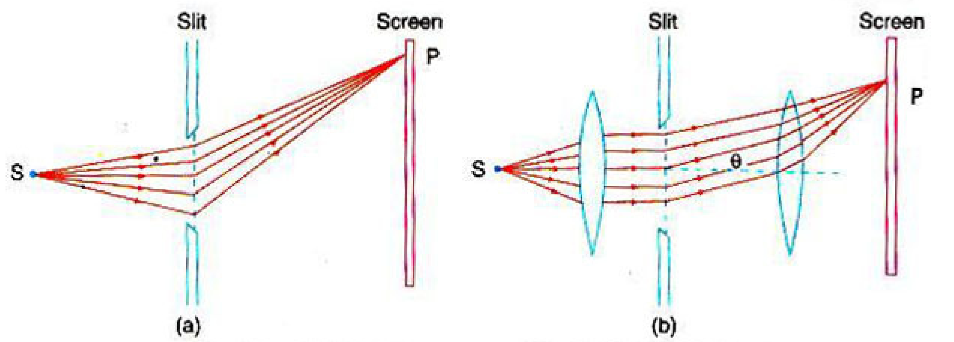
❖ Types of diffraction:

The diffraction phenomenon is classified into **two types**

(i) Fresnel diffraction and (ii) Fraunhoffer diffraction.

(i) Fresnel Diffraction:

In this type of diffraction , the source of light and screen are effectively at finite distances from the obstacle see figure 12a.



Conditions for Fresnel diffraction and Fraunhoffer diffraction.

Figure 12

Observation of Fresnel diffraction phenomenon does not require any lenses. The incident wave front is not planner. As a result, the phase of secondary wavelets is not the same at all points in the plane of the obstacle. The resultant amplitude at any point of the screen is obtained by the mutual interference of secondary wavelets from different elements of unblocked portions of wavefront. It is experimentally simple but the analysis proves to be very complex.

(ii) Fraunhoffer's diffraction

In this type of diffraction, the source of light and the screen are effectively at infinite distances from the obstacle. Fraunhoffer diffraction pattern can be easily observed in practice. The conditions required for Fraunhoffer diffraction are achieved using two convex lenses , one to make the light from the source parallel and the other to focus the light after diffraction on to the screen see figure 12 b, The diffraction is thus produced by the interference between parallel rays. The incident wave front as such is plane and the secondary wavelets, which originate from the unblocked portions of the wave front, are in the same phase at every point in the plane of the obstacle. This problem is simple to handle mathematically because the rays are parallel. The incoming light is rendered parallel with a lens and diffracted beam is focused on the screen with another lens.

❖ Diffraction due to a Narrow Wire:

In Fig.13, S is a narrow slit illuminated by monochromatic light, AB is the diameter of the narrow wire and MN is the screen. The length of the wire is parallel to the illuminated slit and perpendicular to the plane of the paper. The screen is also perpendicular to the plane of the paper. XY is the incident cylindrical wave front. P is a point on the screen such that SOP is perpendicular to the screen. EF is the region of the geometrical shadow and above E and below F, the screen is illuminated.

Now let us consider a point P' on the screen in the illuminated portion. Let us join S to O' , a point on the wave front . O' is the pole of the wave front with reference to P'. the intensity P' due to the wave front above O' is the same at all points and the effect due to the wave front BY s negligible. The intensity at P' will be a maximum or a minimum depending on whether the number of half period strips between O' and A is odd or even. Thus , in the illuminated portion of the screen, diffraction bands of gradually diminishing intensity will be observed. The distinction between maxima and minima will become less if P' is far away from the edge E of the geometrical shadow. Maxima and minima can not be distinguished if the wire is very narrow, because in that case the portion BY of the wavefront also produces illumination at P.

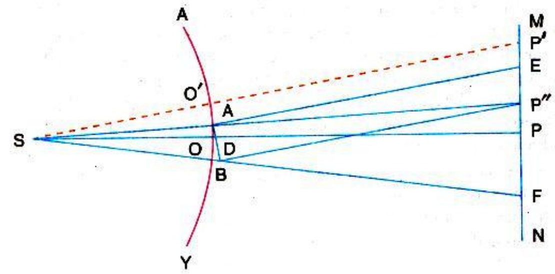


Figure 13

Now consider a point P" in the region of the geometrical shadow. Interference bands of equal width will be observed in this region due to the fact that points A and B, of the incident wavefront , are similar to two coherent sources . The point P" will be of maximum or minimum intensity , depending on whether the path difference (BP" –AP") is equal to even or odd multiples of $\lambda/2$. The fringes width $\beta$is given by

$$\beta = D\ \lambda/d$$

Where D is the distance between the wire and the screen, λ is the wave length of the light and d is the distance between the two coherent sources. In this case d=2r where 2r is diameter of the wire (AB = 2r).

$$\beta = D\lambda/d \quad \ldots\ldots\ldots\ (a)$$
$$\beta = D\lambda/2r \quad \ldots\ldots\ldots\ (b)$$
$$r = D\lambda/2\beta \quad \ldots\ldots\ldots\ (c)$$
$$\lambda = 2\beta r / D \quad \ldots\ldots\ldots\ (d)$$

Here β, is the fringe width corresponds to the distance between any two consecutive maxima. Thus, from equation (c) and (d), knowing the values of r or λ, λ or r can be determined . In figure 14 bands marked "a" represents the interference bands in the region of the geometrical shadow, bands marked "b" and "c" represents the diffraction bands in the illuminated portion. The intensity distribution due to a narrow wire is shown in figure 15(a). The centre of the geometrical shadow is bright.

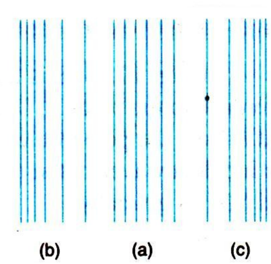


Figure 14

On the other hand if the wire is very thick , the interference bands cannot be noticed. Now from equation (b), $\beta = D\lambda/2r$; where β is the fringe width. As the diameter of the wire increases the fringe width decreases and if the wire is sufficiently thick, the width of the interference fringes decreases considerably and they cannot be distinguished. The intensity falls off rapidly in the geometrical shadow.

The diffraction pattern in the illuminated portion will be similar to that of the thon wire as in figure 15(b). Coloured fringes will be observed with white light.

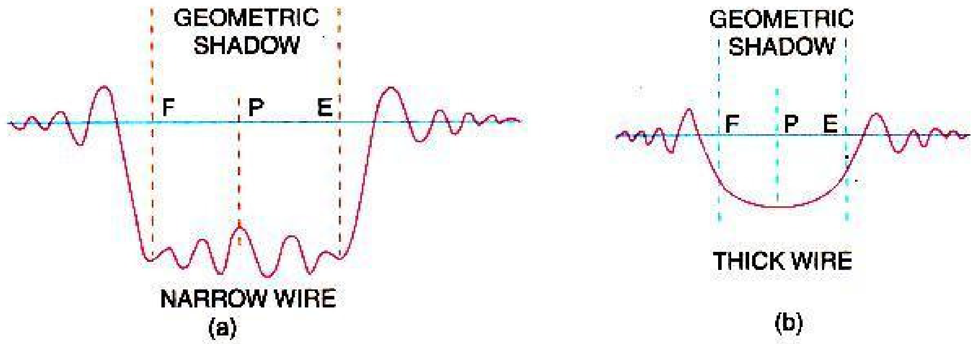


Figure 15

❖ Cornu's Spiral:

To find the effect at a point due to an incident wave front Fresnel's method consists in dividing the wavefront into half period strips or half period zones. The path difference between secondary waves from two corresponding points of neighboring zones is equal to $\lambda/2$.

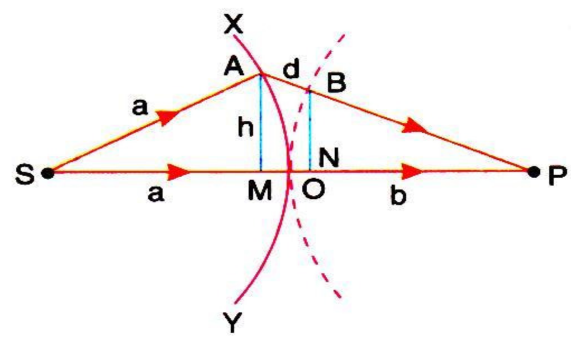


Fig.16

In figure 16, S is a point source of light and XY is the incident spherical wave front. With reference to the point P, O is the pole of the wave front. Let a and b be the distances of the points and P from the pole of the wave front. With p as centre and b, the radius, let us draw a sphere touching the incident wavefront at O. the path difference between the waves traveling in the directions SAP and SOP is given by

$$d = SA + AP - SOP = SA + AP - (SO+OP) = a+AB+b-(a+b) = AB$$

For large distances of a and b , AM and BN can be taken to be approximately equal and the path difference d can be written as

$$d = AB = MO+ON$$

But , from the property of a circle,

$$MO = \frac{AM^2}{2SO} = \frac{h^2}{2a} \quad and \quad ON = \frac{BN^2}{2OP} = \frac{h^2}{2b} \quad approximately$$

$$d = \frac{h^2}{2a} + \frac{h^2}{2b} = \frac{h^2}{2ab}(a+b) \; . \tag{28}$$

If AM happens to be the radius of the nth half period zone, the this path difference is equal to $\frac{n\lambda}{2}$ according to the Fresnel's method of constructing the half period zones.

$$\frac{h^2}{2ab}(a+b) = \frac{n\lambda}{2} \tag{29}$$

The resultant amplitude at an external point due to the wave front can be obtained by the following method. Let the first half period strip of the Fresnel's zone be divided in to eight sub strips, and these vectors are represented from O to M_1(See figure 16a). The continuous phase change is due to the continuous increase in the obliquity factor from O to M_1.The resultant amplitude at the external point due to the first half period strip is given by $OM_1 = m_1$, Similarly , if the process is continued , we obtain the vibration curve M_1M_2 . The portion M_1M_2 corresponds to the second half period strip.

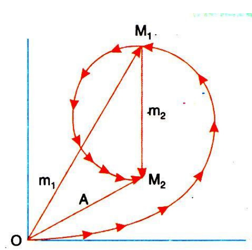


Figure 16a

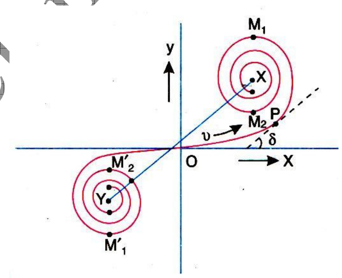


Figure 16b

The resultant amplitude at the point due to the first two half period strips is given by $OM_2 = A$. if instead of eight sub strips, each period zone is divided into sub strips of infinitesimal width , a smooth curve will be obtained. The complete vibration curve for whole wave front will be a spiral as shown in figure 16b.

X and Y corresponds to the two extremities of the wave front and M_1 and M_2 refer to the edge of the first, second, etc. Half period strips of the lower portion of the wave front. This is called Cornu's Spiral. The characteristics of this curve is that for any point P on the curve, the phase lag δ directly proportional to the square of the v. The distance is measured along the length of the curve from the point O. For oath difference of λ , the phase difference 2π. Hence, for a path difference of d , the phase difference δ is given by

$$\delta = \frac{2\pi}{\lambda} d \tag{30}$$

Substituting the value of d from equation 28, we get ,

$$\delta = \frac{\pi}{2}\left[\frac{2h^2(a+b)}{ab\lambda}\right] \tag{31}$$

$$\delta = \frac{\pi}{2}v^2 \tag{32}$$

The value of v is given by

$$v^2 = \frac{2h^2(a+b)}{ab\lambda} \qquad \text{or} \qquad v = h\sqrt{\frac{2(a+b)}{ab\lambda}} \tag{33}$$

Cornu's Spiral can be used for any diffraction problem irrespective of the values of a ,b and λ.

❖ Fraunhoffer Diffraction

We have already seen about the Fraunhoffer diffraction and we also know that in this type of diffraction, the source of light and the screen are effectively at infinite distances from the obstacle. And we have also seen that this problem is simple to handle mathematically because the rays are parallel. The incoming light is rendered parallel with a lens and diffracted beam is focused on the screen with another lens.

❖ Fraunhoffer Diffraction at Double Slit:

In figure 17, AB and CD are two rectangular slits parallel to one another and perpendicular to the plane of the paper. The width of each slit is a and the width of opaque portion is b. L is a collecting lens and MN is a screen perpendicular to the plane of paper. P is a point on the screen such that OP is perpendicular to the screen. Let a plane wave front be incident on the surface of XY. All the secondary waves traveling in a direction parallel to OP come to focus at P. Therefore, P, corresponds to the position of the central bright maxima,

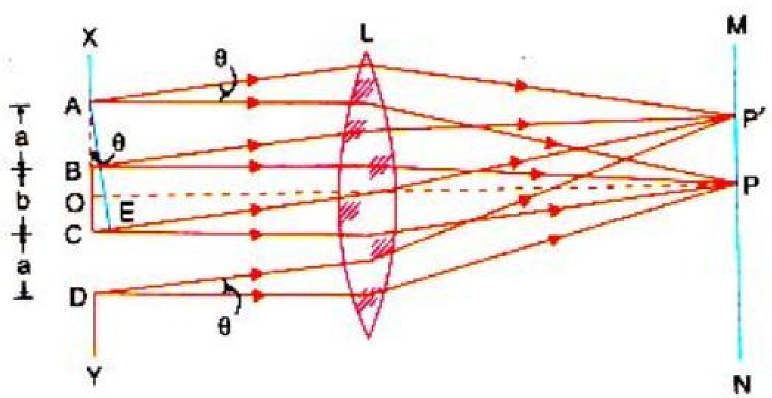


Figure 17

In this case the diffraction pattern has to be considered in to two parts (i) the interference phenomenon due to the secondary wave emanating from the corresponding points of the two slits and (ii) the diffraction pattern due to the secondary waves from the two slits individually. For calculating the position of the interference maxima and minima, the diffracting angle is denoted as θ and for the diffraction maxima and minima it is denoted as ϕ . Both the angle θ and ϕ refer to the angle between direction of the secondary wave and the initial direction of the incident light.

(i) Interference maxima and minima:

Let us consider the secondary waves traveling in a direction inclined at an angle θ with the initial direction.

In the ΔCAN (see figure 18) ,

$$\sin\theta = \frac{CN}{AC} = \frac{CN}{a+b}$$

$$\therefore \quad CN = (a+b)Sin\theta \qquad (34)$$

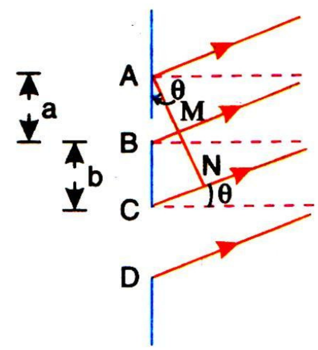


Figure 18

If the path difference is equal to odd multiples of λ/2 , θ gives the direction of minima due to interference of the secondary waves from the two slits.

$$\therefore CN = (a+b)\sin\theta_n = (2n+1)\frac{\lambda}{2} \qquad (35)$$

Now putting n=1,2,3,....etc. the values of θ_1, θ_2, θ_3......etc. corresponding to the directions of minima can be obtained. From equation 2

$$\sin\theta_n = \frac{(2n+1)}{2(a+b)}\lambda \qquad (32)$$

On the other hand , if the secondary waves travel in a direction θ' such that the path difference is even multiples of λ/2 , then θ' gives the direction of the maxima due to interference of light waves emanating from the two slits.

$$\therefore CN = (a+b)\sin\theta'_n = 2n\frac{\lambda}{2}$$

Or

$$\sin\theta'_n = \frac{n\lambda}{(a+b)} \qquad (33)$$

putting n=1,2,3,....etc. the values of θ'_1, θ'_2, θ'_3......etc. corresponding to the directions of maxima can be obtained. From equation (32), we get

$$\sin\theta_1 = \frac{3\lambda}{2(a+b)} \qquad \text{and} \qquad \sin\theta_2 = \frac{5\lambda}{2(a+b)}$$

$$\therefore \sin\theta_2 - \sin\theta_1 = \frac{\lambda}{(a+b)} \qquad (34)$$

Thus the angular separation between any two consecutive minima or maxima is equal to $\frac{\lambda}{(a+b)}$. The angular separation is inversely proportional to (a+b) , the distance between the two slits.

(ii) Diffraction Maxima and Minima:

Let us consider the secondary waves traveling in a direction inclined at an angle Φ with initial direction of the incident light. If the path difference is BM is equal to λ the wave length of light used then Φ will give the direction of the minimum see figure 20. That is, the path difference between secondary waves emanating from the extremities of a slit(i.e. points A & B) is equal to λ. Considering the wave front on AB to be made up of the two halves, the path difference between the corresponding points of the upper and lower halves is equal to λ /2.

The effect at P' due to the wave front incident on AB is zero. Similarly for the same direction of the secondary waves , the effect at P' due to the wave front incident on the slit CD is also zero. In general $a \sin \Phi_n = n\lambda$.

Putting n=1,2,3,….etc. the values of Φ_1, Φ_2, Φ_3……etc. corresponding to the directions of diffraction minima can be obtained.